Simple Harmonic Motion Review

1. Oscillatory motion where there is a restoring force on the object which is proportional to the displacement of the object
   ex: mass on a spring and a simple pendulum

2. pendulum: the length of the string + the gravitational field (g)
   $T_p = 2\pi\sqrt{\frac{l}{g}}$

   mass on spring: the mass + spring constant $T_s = 2\pi\sqrt{\frac{m}{k}}$

3. B/c the restoring force isn't proportional to displacement

4. a) $T = 2\pi\sqrt{\frac{l}{g}}$
   $T = 2\pi\sqrt{\frac{.8m}{10 m/s^2}}$
   $\boxed{T = 1.8s}$

   b) $t = \frac{1}{4}T$
   $\boxed{t = .45s}$

   c) $U_g = K'$
   $\cancel{m}gh = \frac{1}{2}\cancel{m}v^2$
   $\sqrt{2gh} = v$
   $V = \sqrt{2(10)(.2)}$
   $\boxed{V = 2 m/s}$

   d) It would not affect any of them

5. a) $T = 2\pi\sqrt{\frac{m}{k}}$
   $T = 2\pi\sqrt{\frac{3}{100}}$
   $\boxed{T = 1.1s}$

   b) $t = \frac{1}{4}T$
   $\boxed{t = .275s}$

   c) $U_s = K'$
   $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$
   $\sqrt{\frac{kx^2}{m}} = V$
   $V = \sqrt{\frac{100(.2)^2}{3}}$
   $\boxed{V = 1.2 m/s}$

   d) ans to a) + b) would increase by $\sqrt{2}$, ans to c) would decrease by $1/\sqrt{2}$

   e) $T = 2\pi\sqrt{\frac{m}{k}}$

   $\frac{T}{2\pi} = \sqrt{\frac{m}{k}}$

   $\frac{T^2}{4\pi^2} = \frac{m}{k}$

   $T^2 k = 4\pi^2 m$

   $k = \frac{4\pi^2 m}{T^2}$

   $k = \frac{4\pi^2(3)}{(.25)^2}$

   $\boxed{k = 1900 N/m}$

   $f = 4 Hz$
   $T = .25s$

⑥ a) $T = 2\pi\sqrt{\frac{7}{1000}}$

$T = .53s$

$\boxed{t_{stop} = \frac{1}{4}T = .13s}$

b) $K' = U_s''$

$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$

$\sqrt{\frac{mv^2}{k}} = x$

$\sqrt{\frac{7(10)^2}{1000}} = x$

$\boxed{x = .84m}$

$P_{2kg} = P'_{7kg}$

$2(35) = (2+5)v'$

$v' = 10\ m/s$

c) $x = A\cos\left(\frac{2\pi}{T}t\right)$

$x = (.84m)\cos\left[\left(\frac{2\pi}{.53}\right)(1)\right]$

$\boxed{x = .64m}$

d) $a = \frac{\Sigma F}{m} = \frac{-kx}{m}$ $(x = .84m = A)$

$a = \frac{1000\ N/m\,(.84m)}{7kg}$

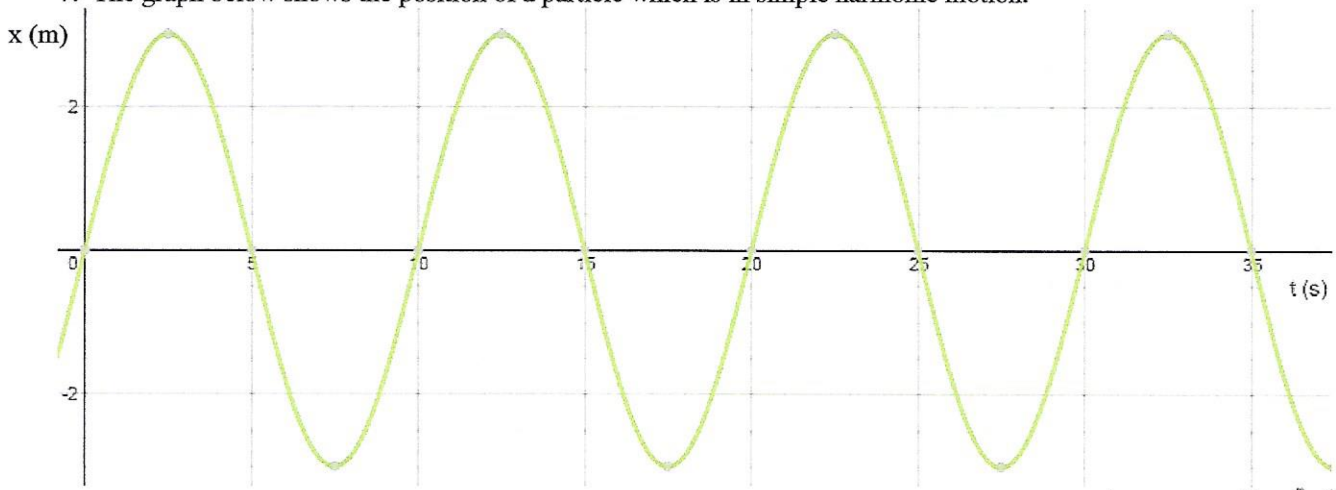
$\boxed{a = 120\ m/s^2}$

e) $\Delta K = \frac{1}{2}(7)(10)^2 - \frac{1}{2}(2)(35)^2$

$\Delta K = 350\ J - 1{,}225\ J$

$\Delta K = -875\ J$ $\boxed{(875\ J\text{ of energy lost})}$

7. The graph below shows the position of a particle which is in simple harmonic motion.



a) What is the period of the motion of this particle? $T = 10s$

b) At what time will the particle first come to rest? $t = 2.5s$

c) At what time will the particle first reach its maximum negative velocity? $t = 5s$

d) At what time will the particle first reach its maximum positive acceleration? $t = 7.5s$

e) At what time does the particle first have its maximum kinetic energy? $t = 0s$

f) $k = \frac{4\pi^2(2)}{10^2}$

$\boxed{k = .79 \, N/m}$

g) $v = \sqrt{\frac{kx^2}{m}}$

$\boxed{V = 1.6 \, m/s}$

h) $a = \frac{kx}{m}$

$a = 1 \, m/s^2$

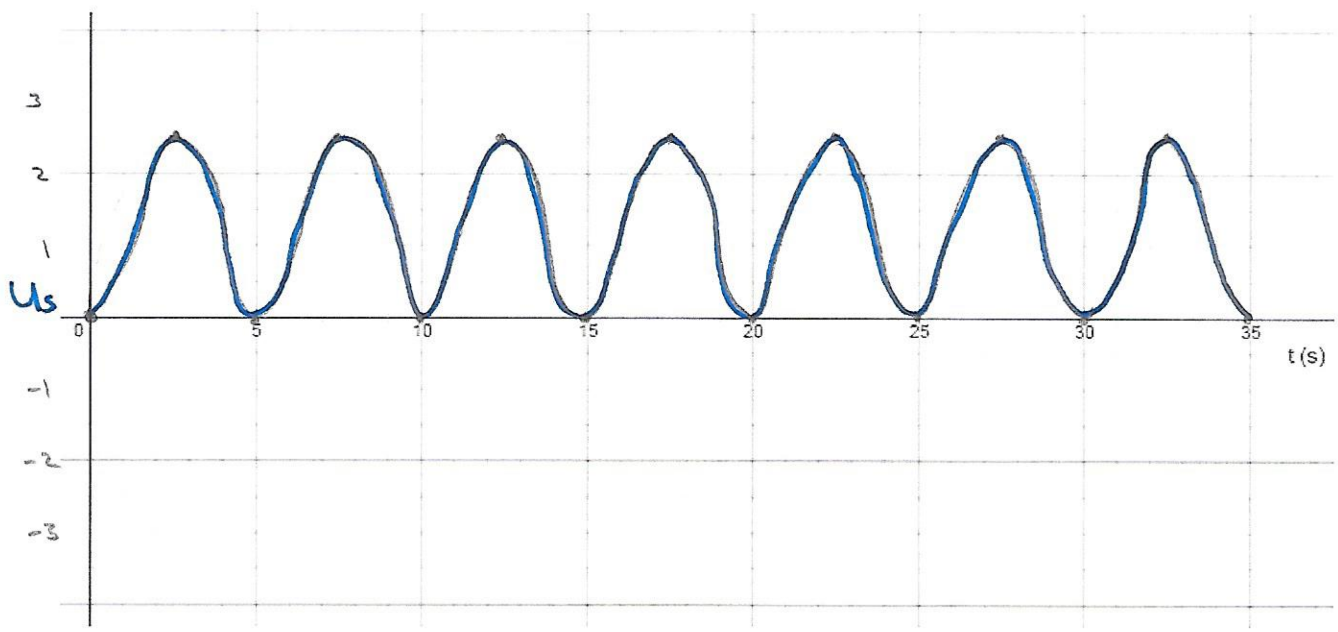
Assume that the particle is a mass of 2 kg which is oscillating on the end of an ideal spring. Calculate:

f) the spring constant of the spring

g) the magnitude of the maximum velocity of the mass

h) the magnitude of the maximum acceleration of the mass

On the graph below, plot the ~~kinetic energy~~, potential energy, and ~~total mechanical energy~~ of the particle.



$U_{s,max} = \frac{1}{2} k x_{max}^2 = \frac{1}{2}(.79)(2.5)^2$

$U_{s,max} = 2.5 \text{ J} = K_{max}$