

1. Two objects have a mass m and are separated by a distance d . The gravitational force between them is F . For each of the following changes, determine how the force of gravity changes.

a) One of the masses is doubled

$$2F \Rightarrow \frac{G(2m)m}{d^2}$$

$$F = \frac{Gmm}{d^2}$$

b) Both of the masses are doubled

$$4F \Rightarrow \frac{G(2m)(2m)}{d^2}$$

c) The distance between them is doubled

$$\frac{1}{4}F \Rightarrow \frac{Gmm}{(2d)^2} = \frac{1}{4} \frac{Gmm}{d^2}$$

d) The distance between them is tripled and one mass is doubled

$$\frac{2}{9}F \Rightarrow \frac{G(2m)m}{(3d)^2} = \frac{2}{9} \frac{Gmm}{d^2}$$

e) The distance between them is halved and both masses are doubled

$$16F \Rightarrow \frac{G(2m)(2m)}{(\frac{1}{2}d)^2} = \frac{4}{\frac{1}{4}} \frac{Gmm}{d^2} = 16 \frac{Gmm}{d^2}$$

2. A satellite of mass 1000 kg is in a circular orbit around a planet with mass 7.5×10^{26} kg at a radius of 9.7×10^7 m.

a) What is the force acting on the satellite?

$$F = \frac{Gmm}{r^2} = \frac{6.67 \times 10^{-11} (7.5 \times 10^{26}) (1000)}{(9.7 \times 10^7)^2} = \boxed{5316 \text{ N}}$$

b) What is the acceleration of the satellite?

$$g = \frac{GM}{r^2} \quad \text{or} \quad g = \frac{F}{m} \quad g = \frac{5316 \text{ N}}{1000 \text{ kg}} = \boxed{5.3 \text{ m/s}^2}$$

c) What is the velocity of the satellite?

→ THE ACCELERATION DUE TO GRAVITY IS CENTRIPETAL ACCELERATION

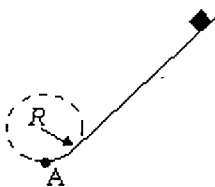
$$\Sigma F = ma \quad \rightarrow \quad mg = \frac{mv^2}{r}$$

$$v = \sqrt{gr} = \boxed{22,673 \text{ m/s}}$$

d) How would the answers to a), b), and c) change if the satellite's mass was doubled?

A → DOUBLE ⇒ FORCE IS PROPORTIONAL TO MASS | B → NO CHANGE | C → NO CHANGE

3. A 25-kg box is sliding down an ice-covered hill. When it reaches point A, the box is moving at 11 m/s. Point A is at the bottom of a circular arc that has a radius $R = 7.5$ m. What is the magnitude of the normal force on the box at Point A?



$$\Sigma F = F_N - mg$$

$$\frac{mv^2}{r} = F_N - mg \quad F_N = \frac{mv^2}{r} + mg$$

$$= \frac{25(11)^2}{7.5} + 25(10) = 400 + 250$$

A) 250 N B) 280 N C) 400 N D) 650 N E) 900 N

Explain how you arrived at your answer:

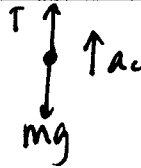
- THE MASS IS MOVING IN A CIRCULAR PATH AT THE BOTTOM OF THE HILL AND EXPERIENCES CENTRIPETAL ACCELERATION UP, TOWARD THE CENTER OF THE CIRCLE. THE NORMAL FORCE IS 1) SUPPORTING GRAVITY & 2) PROVIDING FORCE TO ACCELERATE THE MASS ⇒ IT IS BIGGER THAN JUST WEIGHT.

4. A 0.75-kg ball is attached to a 1.0-m rope and whirled in a vertical circle. The rope will break when the tension exceeds 450 N. What is the maximum speed the ball can have at the bottom of the circle without breaking the rope?

- A) 24 m/s B) 12 m/s C) 32 m/s D) 16 m/s E) 38 m/s

Explain how you arrived at your answer:

THE MAX TENSION IS 450 N.
IN A VERTICAL CIRCLE, MAX
TENSION IS AT THE BOTTOM
BECAUSE TENSION IS OPPOSITE
WEIGHT



$$ma_c = T - mg$$

$$m \frac{v^2}{r} = T - mg$$

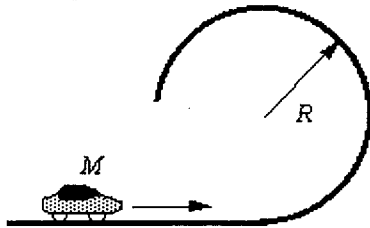
$$.75 \frac{v^2}{1\text{m}} = 450 - .75(10)$$

$$v = 24\text{m/s}$$

Use the following to answer questions 5-6:

A small car of mass M travels along a straight, horizontal track.

As suggested in the figure, the track then bends into a vertical circle of radius R .



5. What is the minimum acceleration that the car must have at the top of the track if it is to remain in contact with the track?

- A) 4.9 m/s^2 , downward
B) 4.9 m/s^2 , upward
C) 9.8 m/s^2 , upward
D) 9.8 m/s^2 , downward
E) 19.6 m/s^2 , upward

Explain how you arrived at your answer:

- THE ACCELERATION OF ANY OBJECT MOVING IN A CIRCLE IS TOWARD THE CENTER — SO IT WILL HAVE A DOWNWARD ACCELERATION AT THE TOP.
- AT THE MINIMUM SPEED, TENSION = 0 AND GRAVITY IS THE ONLY FORCE ACTING

$$ma_c = mg - F$$

$$ma_c = mg$$

$$a_c = g$$

6. Which expression determines the minimum speed that the car must have at the top of the track if it is to remain in contact with the track?

- A) $v = MgR$ B) $v = 2gR$ C) $v^2 = 2gR$ D) $v^2 = gR$ E) $v = gR$

Explain how you arrived at your answer:

USING THE LAST EXPRESSION FROM #5 (WHERE THE $a_c = g$)

$$\frac{v^2}{r} = g$$

$$v^2 = rg$$

7. A 1500-kg car travels at a constant speed of 22 m/s around a circular track that has a radius of 85 m. Which statement is true concerning this car?

- A) The velocity of the car is changing.
 B) The car is characterized by constant velocity.
 C) The car is characterized by constant acceleration.
 D) The car has a velocity vector that points along the radius of the circle.
 E) The car has an acceleration vector that is tangent to the circle at all times.

Explain how you arrived at your answer:

• AS IT MOVES IN A CIRCLE W/ CONSTANT SPEED - THE DIRECTION OF THE CAR CHANGES → MEANING THAT THE VELOCITY CHANGES. VELOCITY ALWAYS POINTS TANGENT TO THE CIRCLE AND ACCELERATION POINTS ALONG THE RADIUS.

8. A car traveling at 20 m/s rounds a curve so that its centripetal acceleration is 5 m/s^2 . What is the radius of the curve?

- A) 4 m B) 8 m C) 80 m D) 160 m E) 640 m

$$a_c = \frac{v^2}{r} \quad r = \frac{v^2}{a} = \frac{20^2}{5}$$

9. A certain string just breaks when it is under 400 N of tension. A boy uses this string to whirl a 10-kg stone in a horizontal circle of radius 10 m. The boy continuously increases the speed of the stone. At approximately what speed will the string break?

- A) 10 m/s B) 20 m/s C) 80 m/s D) 100 m/s E) 400 m/s

Explain how you arrived at your answer:

IN A HORIZONTAL CIRCLE, THE ONLY FORCE ACTING ON THE OBJECT IS TENSION.

THE BIGGEST TENSION CAN BE IS 400 N.

$$\Sigma F = T \quad \Rightarrow \quad \frac{mv^2}{r} = T \quad \Rightarrow \quad v = \sqrt{\frac{T \cdot r}{m}} = \underline{20 \text{ m/s}}$$

10. Which force is responsible for holding a car in a curve?

- A) the car's weight
 B) the force of friction
 C) the reaction force to the car's weight
 D) the vertical component of the normal force
 E) the horizontal component of the normal force

Explain how you arrived at your answer:

• WITHOUT FRICTION, THE CAR WOULD CONTINUE MOVING IN A STRAIGHT LINE (THINK OF DRIVING ON ICE).
 - FRICTION PUSHES THE CAR TOWARD THE CENTER OF THE TURN.

11. You are spinning a bucket full of water (mass = 2 kg) by a rope (of L = 1 m) in a vertical circle.

a. What is the minimum speed at which you can swing the bucket without getting wet?

@ MIN. SPEED, $T=0$ AND $m_{acc} = mg$
 WEIGHT CAUSES CENTRIPETAL ACCELERATION $\frac{v^2}{r} = g$ $v = \sqrt{rg} = \sqrt{1m(10m/s^2)}$
 $\approx 3.16 m/s$

b. The rope will provide up to 200 N of force before it breaks. What is the maximum speed at which you can swing the bucket?

• THE ROPE BREAKS AT THE BOTTOM
 • TENSION IS 200 N

$MA = T - mg$
 $2kg \frac{v^2}{1m} = 200N - 20N$
 $V = 9.5 m/s$

c. If the rope does snap when the bucket is at its lowest point, in which direction does the bucket fly? Assume that you started spinning the bucket like you would an underhanded throw.

• THE BUCKET WILL CONTINUE MOVING IN A STRAIGHT LINE - HORIZ. TO EARTH.

12. A satellite of mass m_S orbits a planet of mass m_P at an altitude equal to twice the radius (R) of the planet.

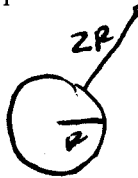
What is the satellite's speed assuming a perfectly circular orbit?

(A) $v = \sqrt{\frac{Gm_P}{R}}$

(C) $v = \sqrt{\frac{Gm_S}{2R}}$

(B) $v = \sqrt{\frac{Gm_S}{R}}$

(D) $v = \sqrt{\frac{Gm_P}{3R}}$



← MASS OF THE SATELLITE GOES AWAY.

Explain how you arrived at your answer:

• WITH AN ALTITUDE OF $2R$ - THE SATELLITE IS $3R$ FROM THE CENTER OF THE PLANET.
 • CENTRIPETAL FORCE IS EQUAL TO GRAVITATIONAL FORCE

$\frac{m v^2}{r} = \frac{G M m}{r^2}$
 $v = \sqrt{\frac{GM}{3R}}$

13. Which of the following changes would increase the magnitude of the gravitational field intensity an object feels when near a planet? (Select two answers.)

B
D

- (A) increase the mass of the object
- (B) increase the mass of the planet
- (C) decrease the spin rate of the planet
- (D) decrease the separation distance between object and planet

Explain how you arrived at your answer:

$g = \frac{F_g}{m}$

• GRAVITATIONAL FIELD IS ACCELERATION DUE TO GRAVITY

• TO INCREASE g ,
 $m_p \uparrow, r \downarrow$

$g = \frac{GM}{r^2}$

• IT DOES NOT DEPEND ON THE MASS OF THE OBJECT - JUST THE PLANET

14. The gravitational field at the location of the International Space Station is about 9 m/s^2 . Explain why, even though it is in space, the ISS has 90% of our gravity.

THE ISS IS ONLY SLIGHTLY FURTHER AWAY FROM THE CENTER OF THE EARTH THAN WE ARE. $g = \frac{GM}{r^2}$ ← IF r INCREASES BY A SMALL AMOUNT, g DECREASES BY A SMALL AMOUNT.

15. The gravitational field at the location of the International Space Station is about 9 m/s^2 . Explain why astronauts in the ISS feel weightless.

THE ASTRONAUTS ARE IN FREE FALL. WHILE OBJECTS IN FREE FALL EXPERIENCE WEIGHT (THEY DO ACCELERATE BECAUSE OF IT), THEY DON'T FEEL IT.

16. Explain why the orbital speed of a satellite does not depend on the mass of the satellite.

- MATHEMATICALLY, ~~THEY~~ ~~OF~~ THE MASS CROSSES OUT

$$\Sigma F = F_g$$

$$m a_c = \frac{GMm}{r^2}$$

- THE CENTRIPETAL FORCE / NET FORCE IS PROPORTIONAL TO THE MASS OF THE OBJECT.
- THE GRAVITATIONAL FORCE DEPENDS ON THE MASS IN THE SAME WAY
- SINCE BOTH FORCES DEPEND ON THE OBJECT'S MASS SIMILARLY, THE MASS "CROSSES OUT" AND DOESN'T AFFECT THE OTHER QUANTITIES.